**Topic**: Double-angle identities

**Question**: Find the values.

If $\theta$ is an angle in the fourth quadrant whose sine is equal to $-4/5$, what are the values of $\sin 2\theta$ and $\cos 2\theta$?

**Answer choices**:

A $\sin 2\theta = \frac{3}{5}$ and $\cos 2\theta = -\frac{4}{5}$

B $\sin 2\theta = -\frac{24}{25}$ and $\cos 2\theta = -\frac{7}{25}$

C $\sin 2\theta = \frac{2\sqrt{6}}{5}$ and $\cos 2\theta = -\frac{1}{5}$

D $\sin 2\theta = \frac{7}{25}$ and $\cos 2\theta = -\frac{24}{25}$

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**Solution**: B

By the double-angle identity for sine,

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

Thus we need to find $\cos \theta$.

By the basic Pythagorean identity,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

Substituting $-4/5$ for $\sin \theta$ gives

$$\cos^2 \theta = 1 - \left(-\frac{4}{5}\right)^2 = 1 - \left(\frac{16}{25}\right) = \frac{9}{25}$$

Since $\theta$ is in the fourth quadrant, $\cos \theta$ is positive, so

$$\cos \theta = \sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

Substituting $\cos \theta = 3/5$ and $\sin \theta = -4/5$ into the double-angle identity for sine, we obtain

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = -\frac{24}{25}$$

By the double-angle identity for cosine,

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$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

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**Topic**: Double-angle identities

**Question**: Which of the following is true?

Let $\theta$ be an angle in the third quadrant whose cosine is equal to $-\sqrt{10}/11$.

**Answer choices**:

A $\cos 2\theta = -\frac{101}{121}$ and $\sin 2\theta = \frac{2\sqrt{1{,}110}}{121}$

B $\cos 2\theta = -\frac{101}{121}$ and $\tan 2\theta = -\frac{\sqrt{1{,}110}}{101}$

C $\tan 2\theta = -\frac{2\sqrt{1{,}110}}{101}$ and $\sin 2\theta = \frac{101}{121}$

D $\sin 2\theta = \frac{1}{121}$ and $\cos 2\theta = -\frac{101}{121}$

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**Solution**: A

By the double-angle identity for cosine,

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

In order to apply this identity, we first have to find $\sin^2\theta$.

By the basic Pythagorean identity,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

Since $\cos\theta = -\sqrt{10}/11$, we get

$$\sin^2\theta = 1 - \left(-\frac{\sqrt{10}}{11}\right)^2 = 1 - \left(\frac{10}{121}\right) = \frac{111}{121}$$

Also,

$$\cos^2\theta = \left(-\frac{\sqrt{10}}{11}\right)^2 = \frac{10}{121}$$

Substituting $\cos^2\theta = 10/121$ and $\sin^2\theta = 111/121$ into the double-angle identity for cosine, we obtain

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{10}{121} - \frac{111}{121} = -\frac{101}{121}$$

To find $\sin 2\theta$ for this same angle $\theta$, we'll use the double-angle identity for sine:

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$$\sin 2\theta = 2\sin\theta\cos\theta$$

We've already found that

$$\sin^2\theta = \frac{111}{121}$$

but we don't yet know the value of $\sin\theta$. Since $\theta$ is in the third quadrant, $\sin\theta$ is negative. Therefore,

$$\sin\theta = -\sqrt{\frac{111}{121}} = -\frac{\sqrt{111}}{\sqrt{121}} = -\frac{\sqrt{111}}{11}$$

Now we're ready to apply the double-angle identity for sine:

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{\sqrt{111}}{11}\right)\left(-\frac{\sqrt{10}}{11}\right) = \frac{2\sqrt{111}(\sqrt{10})}{121} = \frac{2\sqrt{1{,}110}}{121}$$

Thus

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\left(\frac{2\sqrt{1{,}110}}{121}\right)}{\left(-\frac{101}{121}\right)} = -\frac{2\sqrt{1{,}110}}{101}$$

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**Topic**: Double-angle identities

**Question**: Pick the equal value.

$$(\sin - 65^\circ)[2(\cos 65^\circ)]$$

**Answer choices**:

A $\cos 130^\circ$

B $\cos(-65^\circ)$

C $\sin(-65^\circ)$

D $\sin(-130^\circ)$

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**Solution**: D

By the even identity for the cosine function,

$$\cos 65^\circ = \cos(-65^\circ)$$

Therefore,

$$(\sin - 65^\circ)[2\cos 65^\circ] = (\sin - 65^\circ)[2\cos(-65^\circ)]$$

Changing the order of the factors on the right-hand side of this equation, we obtain

$$(\sin - 65^\circ)[2\cos 65^\circ] = 2(\sin - 65^\circ)(\cos - 65^\circ)$$

By the double-angle identity for the sine function,

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Now let $\theta = -65^\circ$. Then

$$\sin[2(-65^\circ)] = 2(\sin - 65^\circ)(\cos - 65^\circ)$$

Simplifying the left-hand side of this equation, we have

$$\sin(-130^\circ) = 2(\sin - 65^\circ)(\cos - 65^\circ)$$

We have shown that the given expression, $(\sin - 65^\circ)[2(\cos 65^\circ)]$, is equal to

$$2(\sin - 65^\circ)(\cos - 65^\circ)$$

and we have applied the double-angle identity for the sine function to show that

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$$2(\sin - 65^\circ)(\cos - 65^\circ) = \sin(-130^\circ)$$

Combining these results, we get

$$(\sin - 65^\circ)[2(\cos - 65^\circ)] = \sin(-130^\circ)$$

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